# MTH330: Problem Set #5

Due: Wednesday 06/20/12

Your calculations must include all arguments, justifications and proofs that are needed. You can only assume and use what we have shown in the lecture and in earlier homeworks or something which is obvious. Only writing numbers will not give you full points!

## Problem 1 *Volumes*

(a) W&A p. 285 no. 29: $ABCD$-$PQRS$ is a cube of side 8. Find the volume of the pyramid $SQCR$. *Remark: The points are denoted counter-clockwise on each level*

(b) W&A p. 285 no. 23: Find the volume and total surface area of a right prism of altitude 12, if its base is a triangle of sides $5, 6$ and 7.

. . . . . . . . .

## Problem 2 *About lines and planes in space*

(a) W&A p. 172 no. 5: A line $x$ is parallel to a plane $M$ and $M$ contains a line $y$. Is it possible that $x$ and $y$ intersect? Ist it possible that $x$ and $y$ are not parallel?

(b) W&A p. 171 no. 16,17,18: In the diagram on page 171 show that $PA \perp AC, SM \perp AC$ and $AC \perp$plane through $SDM$. *Look carefully at the theorems and definitions we have had. Each problem should be a very short proof/argument.*

. . . . . . . . .

## Problem 3 *More volume calculations with a tiny new necessary calculation*

W&A p. 278 no. 13: All the edges of a regular square pyramid have length 8. Find its volume and total surface area.

. . . . . . . . .

## Problem 4 *Another triangle construction*

Suppose you are given the length $b$, the length $c$ and the length $h_a$ of the altitude from $A$ to the opposite side $a$ in a triangle $\triangle ABC$.

(a) Given the data of $b, c$ and $h_a$, describe a way of constructing the triangle $\triangle ABC$. *Hint: One way of doing this is by starting with $A$, drawing $h_a$ and then trying to find $B$ and $C$ by the remaining conditions.*

(b) W&A p. 347 no 11: Do the construction with $c = 2\frac{1}{2}$in, $b = 2$in and $h_a = 1\frac{1}{2}$in.

. . . . . . . . .

## Problem 5 *Euler Characteristics and polyhedrons*

(a) What is the number of vertices, egdes and faces and the Euler characteristic of a regular dodecahedron? Please don't just look it up, try to figure it out yourself.

(b) What is the number of vertices, edges and faces and the Euler characteristic of a regular icosahedron? Please don't just look it up, try to figure it out yourself.

(c) What is the volume of a regular tetrahedron of edge length $a$?

(d) Which figure do you obtain if you join the centers of the faces of a regular tetrahedron?

(e) If the original tetrahedron had edge length $a$ what is the ratio of the volumes of the inner tetrahedron and the original one?

. . . . . . . . .

## Problem 6 *Parallel triangles are similar*

Suppose you are given two triangles $\triangle ABC$ and $\triangle XYZ$ whose corresponding sides are pairwise parallel.

(a) Forget about one pair of lines for a moment. The remaining figure consists of two angles whose rays are parallel. Show that the angles are equal.

(b) Deduce that the two triangles are similar.

. . . . . . . . .

## Problem 7 *The radius of the Feuerbach circle*

Recall the Feuerbach circle of a triangle. In class we didn't show that its radius was half the radius of the circumcircle of the triangle. This excercise is about proving this result.

(a) Why is the Feuerbach circle the circumcircle of the triangle formed by the three midpoints of the sides of the original triangle? *Hint: This question is essentially trivial.*

(b) What is the relation between the triangle formed by the midpoints of the sides of the triangle and the original one? *Hint: We have essentially shown this a few times already.*

(c) Using the two preceding results prove that the radius of the Feuerbach circle is one-half the radius of the circumcircle of the original triangle. *Hint: You may use the obvious observation that the radius of the circumcircle scales the same way as the sides of the triangle.*

. . . . . . . . .

## Problem 8 *(Bonus) Weeks & Adkins p. 286*

Let $\triangle ABC$ be equilateral. Pick $X$ on $AB$ such that $\frac{AX}{XB} = \frac{1}{2}$, $Y$ on $BC$ such that $\frac{BY}{Yc} = \frac{1}{2}$ and $Z$ on $CA$ such that $\frac{CZ}{ZA} = \frac{1}{2}$. Let $AY$, $BZ$ and $CX$ intersect at $P, Q$ and $R$. Prove that the area of $\triangle PQR$ is one-seventh of the area of $\triangle ABC$.

. . . . . . . . .