MTH 330: Problem Set #4

Summer 2012 – Due: Tue 06/12/12

Problem 1 *Weeks & Adkins*

page 373 no. 13: AB is a diameter of a circle. P, Q are points on the line which is tangent to the circle at B. AP meets the circle at C, and AQ meets the circle at D. Prove $\angle ACD = \angle AQP$.

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Problem 2 *Point of view*

Suppose a 6ft-tall person stands on the surface of the earth which is assumed to be a perfect sphere. How far is the horizon from his eyes, i.e. how far can he see? The radius of the earth is 3960 miles.

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Problem 3 *Interior angles of hexagons*

Prove that in an inscribed hexagon, the alternating interior angles add up to $360\,^\circ$.

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Problem 4 *Quadrilaterals with parallel sides*

In a quadrilateral $ABCD$ assume $\angle A = \angle B$ and $\angle C = \angle D$, where A, B, C, D are the points counter-clockwise. Prove that in this case $AB \| CD$.

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Problem 5 *Constructions*

Suppose you are given a circle K (without its center) and a point P outside the circle.

(a) Describe a way to construct a tangent to the circle K through the given point P outside the circle.

(b) Draw a circle of radius 3in and pick a random point outside the circle. Do your construction and construct a tangent to K through P.

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Problem 6 *Parallel tangents*

Suppose you are given a circle K and a point P outside of the circle. Then there are two tangents to K through P and one line connecting P with the center of the circle O.

(a) Call the points where the two tangents touch the circle A and B. Show $\angle APO = \angle OPB$.

(b) Now suppose you are given a second point Q outside K such that $PO \perp OQ$ and PQ is a tangent of K. Then P and Q have each a second tangent to K trough them. Prove that they are parallel.

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Problem 7 *The Simson line of a point*

In this problem you will show the existance of another interesting line in the triangle, the Simson line of a point (here called P). The individual steps of the proof are actually very easy, so if you follow the instructions you should be able to do it. The sketch is in Weeks & Adkins on page 491. Note that even if you are not able to prove an intermediate step you migth get points for every result you show.

Let P be any point on the circumcircle of the triangle ABC. Then we can draw the perpendiculars of P onto all the three sides of the triangle. Call the foot of the perpendicular on AC M, on AB N and on BC L, see the figure. Then prove the following statements:

(a) $\angle PBL = \angle MAP$

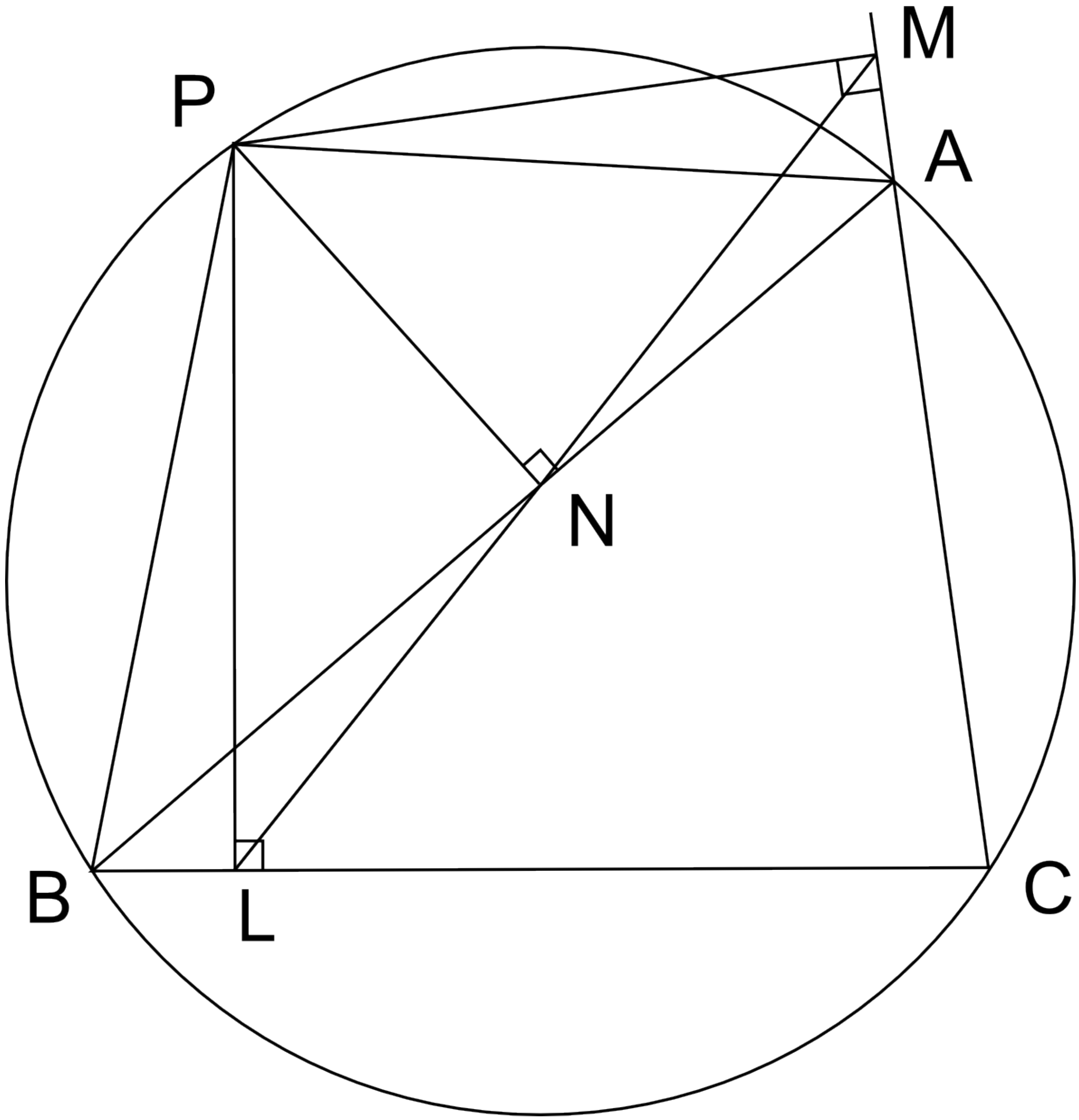
(b) $\angle BPL = \angle APM$

(c) P, B, L, N are on a circle

(d) P, N, A, M are on a circle

(e) $\angle BNL = \angle ANM$ *Hint: I included the last two steps in order to help you find the argument for this step*

(f) L, N and M are collinear *Hint: Although we haven't shown it, you can use some kind of converse of the first lemma we proved at the beginning of the course. It is intuitively clear.*



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Problem 8 *(Bonus question) LMO 490832*

Let $ABCD$ be a parallelogram. On CD pick a point E. The parallel of BD through E intersects BC in a point F and the parallel of AB through F intersects AD in G. The intersection point of FG with BD is called P and the intersection point of the parallel of BD through G with AB is called H. Prove that the quadrilaterals $PFED$ and $PGHB$ have the same area.

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