MTH 330: Problem Set #3

Summer 2012 – Due: Tue 06/05/12

Problem 1 *Wrong proofs*

1. Where is the mistake in the following proof?

   Congruence movements like translations change the area of a triangle as you can see in the following example:

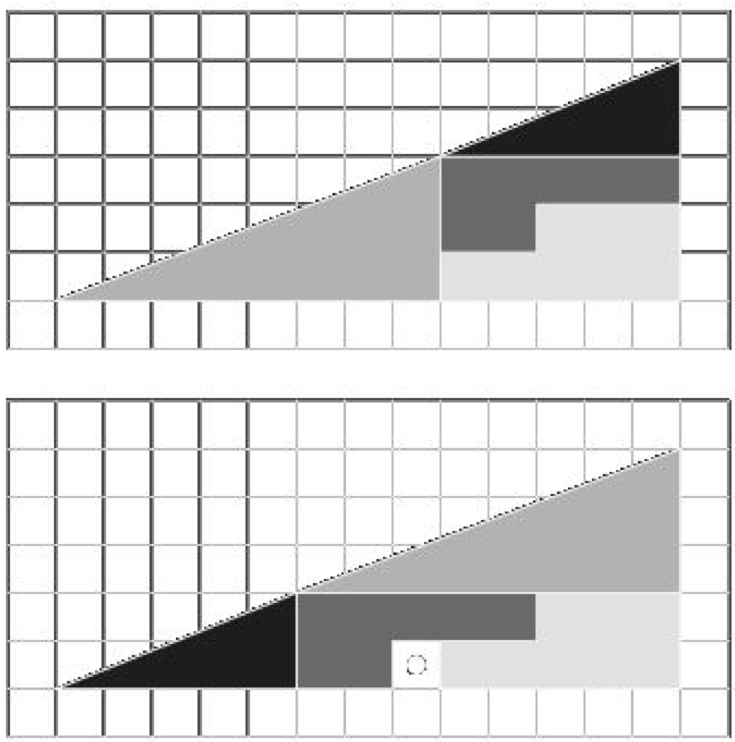

Figure 1: Congruence does not preserve area, this is the counterexample.

2. Where is the mistake in the following proof?

   Take any triangle $\triangle ABC$ and draw the angle bisector of $\angle BAC$ as well as the perpendicular bisector of BC. These two lines intersect in E. Draw the perpendiculars of E to AB and AC and call their intersection points with the sides E and F as in Fig. 2. Then we have $BD = DC$, $\angle EDB = \angle CDE$ and $DE = DE$ and therefore by SAS $\triangle EDB \simeq \triangle EDC$. Because of $\angle FAE = \angle EAG$ by construction, $\angle EFA = \angle AGE$ and therefore $\angle AEF = \angle GEA$ and the shared side AE we have using ASA $\triangle FEA \simeq \triangle GEA$. Using those two congruencies we see $FE = EG$ and $BE = EC$. Since $\angle BFE = \angle EGC = 90^\circ$ we conclude by SSA that $\triangle BFE \simeq \triangle CGE$. This implies $BF = GC$. Since we have from the second congruency $AF = AG$ we conclude $AB = AC$. Therefore, all triangles are isosceles.

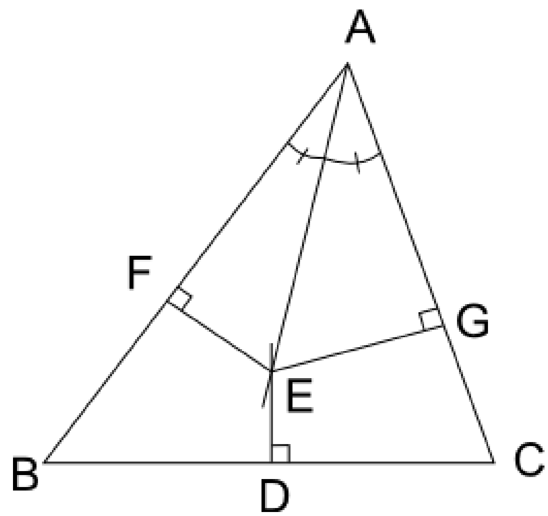

Figure 2: All triangles are isosceles.

. . . .

Problem 2 *Areas of triangles*

In the trapezoid $ABCD$ (where A, B, C and D are the vertices counterclockwise), $AB \| DC$ and the diagonals meet in X. Prove that Area($\triangle AXD$) = Area($\triangle BXC$).

. . . .

Problem 3 *Circumscribing quadrilaterals*

In the quadrilateral $ABCD$, $AB = 4$, $BC = 7$, $CD = 4$, $DA = 3$ and the diagonal $AC = 5$. Can $ABCD$ be circumscribed?

. . . .

Problem 4 *Weeks & Adkins*

Solve the following problems:

1. Page 386 number 8: AB is a chord of a circle, and is of length 12in. M is the midpoint of AB. CD is a chord of the circle which passes through M and is of length 13in. Find CM and MD.

2. Page 195 number 11: G is the centroid of $\triangle ABC$ in which $\angle A = 90\,^\circ$, $BC = 18$ inches. Find the length AG.

. . . .

Problem 5 *Hexagon construction*

Weeks & Adkins page 143 problem 15: Construct a regular hexagon of side 5cm using a straightedge and a compass only.

. . . .

Problem 6 *Proofs from the lecture*

Prove the following five statements:

1. If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram.

2. If one side of a quadrilateral is equal and parallel to the opposite side, then the figure is a parallelogram.

3. The diagonals of a rhombus are perpendicular.

4. The diagonals of a rectangle are equal.

5. Two tangents to a circle K in A and B through an exterior point P satisfy $AP = BP$. Do not use the tangent–secant theorem.

. . . .

Problem 7 *A new construction*

Suppose you are given an arbitrary circle in the plane. Can you construct its midpoint by using compass and straightedge only? If yes, give a construction prescription and prove that it is correct. If not, why is it not possible?

. . . .

Problem 8 *Unknown lengths*

D is a point on the side BC of $\triangle ABC$. A line through B parallel to DA meets CA extended at F and a line through C parallel to DA meets BA extended at E. If $\frac{BA}{AE} = \frac{3}{2}$, $AD = DC = 6$ and $\angle ACB = 60^\circ$, find BD, BF and FA.

. . . .

Problem 9 *Unknown lengths in a circle*

From an external point K a tangent KC and a secant KAB are drawn to a circle of radius 5 and center O. $KO = 13$ and $KA = 9$. Find KC, AB and $\frac{CA}{CB}$.

. . . .

Problem 10 *(Bonus question) Weeks & Adkins p. 150*

$\triangle ABC$ has $CA = CB$, $\angle C = 20^\circ$. D is a point on AC such that $BD = DC$. E is a point on BC such that $BE = BA$. Prove that $\angle BDE = 30^\circ$. *Hint: With center B and radius BD construct an arc of a circle meeting BA extended at X and BC at Y. Draw the segments DX and DY.*

. . . .