MTH 330: Problem Set #2

Summer 2012 – Due: Tue 5/29/12

Problem 1 *Wrong proofs*

1. Where is the mistake in the following proof?

   Draw two intersecting circles and call their intersection points A and B. Draw the two diameters of the two circles through A, call their other ends P and Q and connect them with a line segment. This line segment will intersect the two circles in two points H and K. Now, by the theorem of Thales, H and K are points on semi-circles over a diameter and thus AH and AK are perpendicular to PQ. Looking at the triangle AHK we see that its interior sum is bigger than $180\,^\circ$.

2. Where is the mistake in the following proof?

   Let $ABCD$ be a rectangle. Draw a line AE outside of the rectangle with an acute angle to AB and length $AE = AB$. Connect C and E, draw the perpendicular bisectors of CB through H and CE through K. These line will intersect in a point O. Denote the intersection point of the perpendicular to CB with AD G. Note that HG is a perpendicular bisector of AD, too. Draw OA, OE, OC and OD. Then, by SAS $\triangle EKO \simeq \triangle CKO$ and $\triangle DGO \simeq \triangle AGO$. Therefore, $CO = OE$ and $DO = OA$. By construction, $AE = AB = CD$ and therefore, using SSS, $\triangle CDO \simeq \triangle EAO$. Since $\angle ODG = \angle GAO$ and $\angle ODC = \angle EAO$ (by congruence) we get $90\,^\circ = \angle GDC = \angle EAD = 90\,^\circ + \angle EAB > 90\,^\circ$, which is obviously wrong.

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Problem 2 *Euclid's proof of the lemma for isosceles triangles*

Page 84: In the following figure, $AB = AC$, F and G are points on the extensions of AB and AC s.t. $AF = AG$. FC and BG are drawn. Prove, without using the lemma on isosceles triangles or anything thereafter, that $\angle CBA = \angle ACB$.

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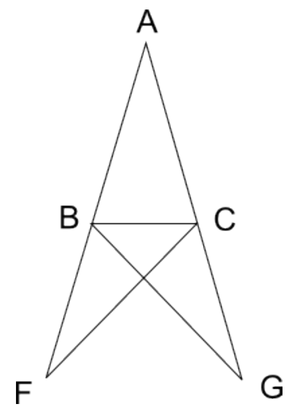
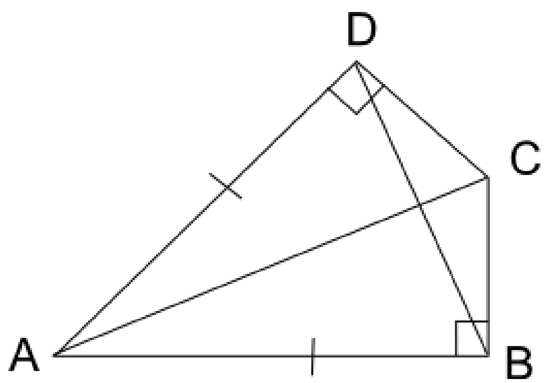


Problem 3 *Page 83 Problem 9*

In the four-sided figure (quadrilateral) $ABCD$, $AB = AD$, $\angle B = \angle D = 90°$. Prove that $AC \perp BD$.



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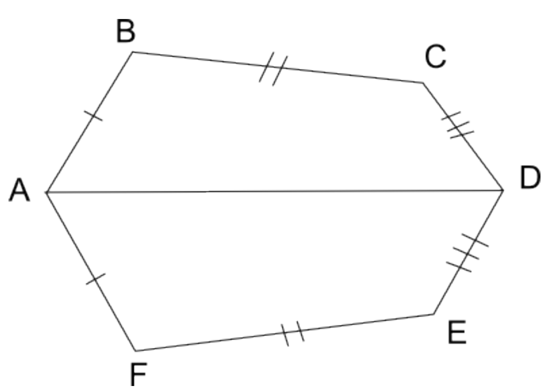
Problem 4 *Page 83 Problem 10*

In the six-sided figure (hexagon) $ABCDEF$, $AB = AF$, $BC = FE$, $CD = DE$, AD bisects $\angle CDE$. Prove $\angle B = \angle F$.



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Problem 5 *Proofs from the lecture*

Prove the following four statements:

1. Let a, b and c be three lines in a plane, $a \| b$ and $b \| c$. Then $a \| c$.
2. Let a, b and c be three lines in a plane, $a \| b$ and c intersects transversally a. Then c intersects transversally b.
3. The cathetus theorem.
4. The right triangle altitude theorem.

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Problem 6 *Little calculations*

Do the following two problems:

1. Weeks & Adkins p. 177 no. 3: When the midpoints of the sides of $\triangle XYZ$ are joined, a triangle of perimeter 15in is formed. What is the perimeter of $\triangle XYZ$?
2. Weeks & Adkins p. 178 no. 5: If AB, CD and EF are parallel, and $AC = \frac{4}{3}CE$, what fractional part is FD of **(a)** DB; **(b)** FB?

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Problem 7 *An easy construction*

Construct a triangle with sides of lengths 6in, 5in and 3in. Construct its three medians by using only pencil, paper, eraser, compass and a straight-edge. What do you notice?

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Problem 8 *Constructions*

In this exercise you are supposed to figure out a way of constructing a triangle if you are only given the lengths of its medians.

1. Consider a triangle $\triangle ABC$. Denote by E the midpoint of AC, by D the midpoint of BC, by O the intersection point of BE and AD (i.e. the three bisectors of the sides or medians), by X the midpoint of AO and by Y the midpoint of BO. Prove that the quadrilateral $DEXY$ is a parallelogram, that $AO = \frac{2}{3}AD$ and that $DY \| CO$. Calculate DY in terms of CO.

2. Give a construction instruction of the triangle $\triangle ABC$ given only the lengths of its medians. *(Hint: Use what you have shown in the first part of the problem. Begin by constructing the triangle $\triangle DOY$)*

3. Construct a triangle whose medians are 5in, 4in and 2.5in, respectively.

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Problem 9 *(Bonus question) LMO 480835*

We assume the following about the five points A, B, C, D and E:

1. The points are on a circle C with center M in this order.
2. M lies on AC.
3. $AB = BC$
4. $CD = DE = EA$

Prove that $\triangle MCD \simeq \triangle MDE \simeq \triangle MEA$ and calculate the interior angles of $\triangle BCD$.

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